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# Towards an improved description of SiDIS by a polarized $^3\text{He}$ target

**Abstract** The possibility of improving the description of the semi-inclusive deep inelastic electron scattering off polarized  $^3\text{He}$ , that provides information on the neutron single spin asymmetries, is illustrated. In particular, the analysis at finite momentum transfers in a Poincaré covariant framework is outlined and a generalized eikonal approach to include final state interaction is presented.

**Keywords** neutron spin asymmetries · Light-front dynamics · final-state interaction · polarized  $^3\text{He}$  target

## 1 Introduction

As it is well known, the quark helicity takes into account at most one third of the proton spin. This motivates the great effort on both experimental and theoretical sides to accurately determine the contributions from the quark orbital angular momentum ( $L_q$ ) and from the gluons. Analogous efforts are carried out to study the neutron. In view of this, the investigations of reactions like the electron semi-inclusive deep inelastic scattering (SiDIS) by a polarized  $^3\text{He}$  target play a very relevant role [1]. Such processes yield information on the so-called transverse-momentum distributions (TMD) of a polarized quark inside a polarized neutron, such that an unprecedented amount of details on the quark dynamics inside the neutron can be achieved. Our aim [2] is to extend a previous analysis of SiDIS by polarized  $^3\text{He}$  target [3], that addressed the extraction of the neutron single-spin asymmetries (SSAs), within a plane wave impulse approximation (PWIA) framework and in the Bjorken limit. One can improve such an approach by: i) dealing with the relativistic effects through a Poincaré covariant description of the nuclear dynamics (see, e.g., [4; 5]) and ii) taking into account the final state interaction (FSI) by adopting [6] the so-called generalized eikonal approach (GEA) (see, e.g. [7]).

## 2 The polarized $^3\text{He}$ nucleus as an effective neutron target

A polarized  $^3\text{He}$  nucleus is an ideal target to study the neutron, since at a 90% level it is equivalent to a polarized neutron. For disentangling the nucleon structure from the dynamical nuclear effects, one can adopt

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an approach based on the spin-dependent spectral function of  ${}^3\text{He}$ ,  $P_{\sigma,\sigma'}(\mathbf{p}, E)$ , (see, e.g. [8]) that yields the probability distribution to find a nucleon with given missing energy, three-momentum and polarization inside the nucleus. By using this formalism, one can safely extract [9] the neutron longitudinal asymmetry,  $A_n$ , from the corresponding  ${}^3\text{He}$  observable,  $A_3^{exp}$ , obtained from the reaction  ${}^3\vec{\text{He}}(\vec{e}, e')X$  in DIS regime, i.e.

$$A_n \simeq (A_3^{exp} - 2p_p f_p A_p^{exp}) / (p_n f_n) \quad (1)$$

with  $p_{n(p)}$  the neutron (proton) effective polarization inside the polarized  ${}^3\text{He}$ , and  $f_{n(p)}$ , the dilution factor. Realistic values of  $p_n$  and  $p_p$  are  $p_p = -0.023$ ,  $p_n = 0.878$  (see, e.g., [9; 3]). In [3], an analogous extraction was applied to the SSA of a transversely polarized  ${}^3\text{He}$  target, obtained from the process  ${}^3\vec{\text{He}}(e, e'\pi)X$ , in order to obtain the SSA of a transversely polarized neutron. In PWIA and adopting the Bjorken limit, the SSAs of  ${}^3\text{He}$  are a convolution of  $P_{\sigma,\sigma'}(\mathbf{p}, E)$ , and the nucleon SSAs, that in turn are convolutions of suitable TMDs and fragmentation functions, phenomenologically describing the hadronization of the hit quark. To improve the previous description, taking into account relativistic effects in the actual experimental kinematics and developing a Poincaré covariant framework for analyzing the SSA of  ${}^3\text{He}$ , one can adopt the Light-Front (LF) Relativistic Hamiltonian Dynamics (RHD), combined with the Bakamjian-Thomas construction of the Poincaré generators. Then, one gets the following expression for the  ${}^3\text{He}$  hadronic tensor [4; 5] (for details see [2])

$$\begin{aligned} \mathcal{W}^{\mu\nu}(Q^2, x_B, z, \tau, \hat{\mathbf{h}}, S_{He}) &\propto \sum_{\sigma,\sigma'} \sum_{\tau} \oint_{\epsilon_S^{min}}^{\epsilon_S^{max}} d\epsilon_S \int_{M_N^2}^{(M_X - M_S)^2} dM_f^2 \\ &\times \int_{\xi_l}^{\xi_u} \frac{d\xi}{\xi^2(1-\xi)(2\pi)^3} \int_{P_\perp^M} \frac{dP_\perp}{\sin\theta} (P^+ + q^+ - h^+) w_{\sigma\sigma'}^{\mu\nu}(\tau, \tilde{\mathbf{q}}, \tilde{\mathbf{h}}, \tilde{\mathbf{P}}) \mathcal{P}_{\sigma'\sigma}^\tau(\tilde{\mathbf{k}}, \epsilon_S, S_{He}) \end{aligned} \quad (2)$$

where  $\tilde{\mathbf{v}} = \{v^+ = v^0 + v^3, \mathbf{v}_\perp\}$ ,  $w_{\sigma\sigma'}^{\mu\nu}(\tau, \tilde{\mathbf{q}}, \tilde{\mathbf{h}}, \tilde{\mathbf{P}})$  is the hadronic nucleon tensor and  $\mathcal{P}_{\sigma'\sigma}^\tau(\tilde{\mathbf{k}}, \epsilon_S, S_{He})$  the LF spin-dependent spectral function, related to the instant-form spectral function,  $\mathcal{S}_{\sigma'_1\sigma_1}^\tau(\mathbf{k}, \epsilon_S, S_{He})$  through the unitary Melosh Rotations,  $D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]$ , as follows

$$\mathcal{P}_{\sigma'\sigma}^\tau(\tilde{\mathbf{k}}, \epsilon_S, S_{He}) \propto \sum_{\sigma_1\sigma'_1} D^{\frac{1}{2}}[\mathcal{R}_M^\dagger(\tilde{\mathbf{k}})]_{\sigma'\sigma'_1} \mathcal{S}_{\sigma'_1\sigma_1}^\tau(\mathbf{k}, \epsilon_S, S_{He}) D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma_1\sigma} \quad (3)$$

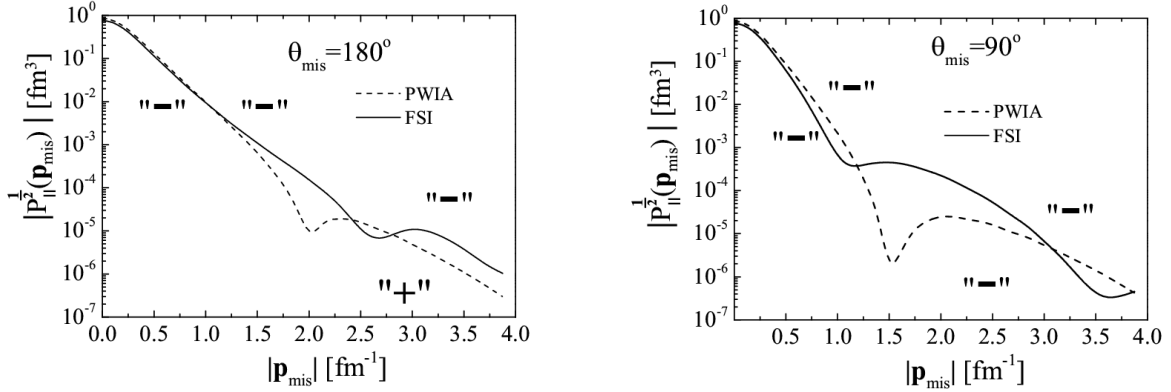
In our approach,  $\mathcal{S}_{\sigma'_1\sigma_1}^\tau$  can be approximated by  $\mathcal{P}_{\sigma'_1\sigma_1}^\tau$ , obtained within a non relativistic framework, since the constraints imposed by the Poincaré covariance can be fully satisfied with our assumptions, namely the LF RHD framework completed by the Bakamjian-Thomas construction [2].

### 3 Beyond PWIA: the generalized eikonal approximation

The second ingredient to be added is GEA (see, e.g., [7] and references therein quoted), devised for taking into account the FSI effects. In particular, FSI effects to be considered are due to the propagation of the debris, formed after the  $\gamma^*$  absorption by a target quark, and the subsequent hadronization, both of them influenced by the presence of a fully-interacting  $(A-1)$  spectator system. Clearly, such FSI effects represent a very complicated many-body problem and their evaluation from first principles is a hard challenge. Therefore it is necessary to develop model approaches for evaluating their impact on the extraction of neutron SSAs. The approximation based on GEA has been recently applied for describing the spectator SiDIS by a polarized  ${}^3\text{He}$  target [6]. The key quantities are the intrinsic overlaps defined as follows

$$\begin{aligned} \mathcal{O}_{\lambda\lambda'}^{\hat{S}_A(FSI)}(\mathbf{p}_N, E) &= \oint d\epsilon_{A-1}^* \rho(\epsilon_{A-1}^*) \langle \hat{S}_{Gl}\{\Phi_{\epsilon_{A-1}^*}, \lambda, \mathbf{p}_N\} | S_A, \Phi_A \rangle \langle S_A, \Phi_A | \hat{S}_{Gl}\{\Phi_{\epsilon_{A-1}^*}, \lambda', \mathbf{p}_N\} \rangle \times \\ &\delta(E + M_A - m_N - M_{A-1}^* - T_{A-1}) \end{aligned} \quad (4)$$

where i)  $E$  is the usual missing energy  $E = \epsilon_{A-1}^* + B_A$ , with  $\epsilon_{A-1}^*$  ( $\rho(\epsilon_{A-1}^*)$ ) the energy (state density) of the spectator system and  $B_A$  the binding energy of the target nucleus, ii)  $\mathbf{p}_N$  the three-momentum of the struck



**Fig. 1** The absolute value  $|\mathcal{P}_{||}^{\frac{1}{2}}|$ , relevant for a spectator SiDIS with a deuteron in the final state, for the reaction  ${}^3\text{He}(\vec{e}, e' {}^2\text{H})X$ , in the Bjorken limit, vs the missing momentum ( $\mathbf{p}_{\text{mis}} \equiv \mathbf{P}_{\text{deut}}$ ), in parallel,  $\theta_{\text{mis}} = 180^\circ$  (left panel), and perpendicular,  $\theta_{\text{mis}} = 90^\circ$  and  $\phi_{\text{mis}} = 180^\circ$  (right panel) kinematics. Dashed line: PWIA. Solid line: FSI effects. The sign of  $\mathcal{P}_{||}^{\frac{1}{2}}$  is indicated by + and -. (After Ref. [6]).

nucleon and iii)  $\hat{S}_{Gl}(1, 2, 3)$  represents the debris-nucleon eikonal scattering S-matrix, that depends upon the relative coordinates only, and it is given by  $\hat{S}_{Gl}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \prod_{i=2,3} [1 - \theta(z_i - z_1) \Gamma(\mathbf{b}_1 - \mathbf{b}_i, z_1 - z_i)]$ , where  $\mathbf{b}_i(z_i)$  is the perpendicular (parallel) component of  $\mathbf{r}_i$  (remind that  $\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 = 0$ ), with respect to the direction of the propagation of the debris  $\mathbf{p}_X$ . In the DIS limit  $\mathbf{p}_X \simeq \mathbf{q}$  ( $\mathbf{q}$  is the three-momentum transfer) and the eikonal S-matrix is defined with respect to  $\mathbf{q}$ . The profile function,  $\Gamma$ , is [7]

$$\Gamma(\mathbf{b}_{1i}, z_{1i}) = \frac{(1 - i\alpha) \sigma_{eff}(z_{1i})}{4\pi b_0^2} \exp\left[-\frac{\mathbf{b}_{1i}^2}{2b_0^2}\right], \quad (5)$$

where  $z_{1i} = z_1 - z_i$ ,  $\mathbf{b}_{1i} = \mathbf{b}_1 - \mathbf{b}_i$ , and  $\sigma_{eff}(z_{1i}) = \sigma_{tot}^{NN}(z_{1i}) + \sigma_{tot}^{\pi N}(z_{1i}) N_{eff}^\pi$ , with  $N_{eff}^\pi$  the effective number of pions which are produced. Indeed, the effective cross sections,  $\sigma_{eff}(z_{1i})$  depends also on the total energy of the debris,  $W^2 \equiv p_X^2 = (p_N + q)^2$ , but such a dependence is weak, if the energy is not too large and the hadronization process develops inside the nuclear environment. Hence, the effective cross section can be approximated as  $\sigma_{eff}(z_{1i}, x_{Bj}, Q^2) \sim \sigma_{eff}(z_{1i})$  [7; 10]. Successful applications of GEA to unpolarized SiDIS can be found in Refs. [7; 11]. In conclusion, the embedding of the GEA overlaps, Eq. (4), in a Poincaré covariant framework can be achieved through a generalization of the relation shown in Eq. (3) [2].

#### 4 Results and conclusions

The impact of the FSI treatment can be appreciated by considering the following quantity, needed in the evaluation of SiDIS processes and strictly related to the nuclear dynamics, namely  $\mathcal{P}_{||}^{\frac{1}{2}}(F_{SI})(\mathbf{p}_N, E)$ , viz

$$\mathcal{P}_{||}^{\frac{1}{2}}(F_{SI})(\mathbf{p}_N, E) = \frac{1}{2} \left[ \mathcal{O}_{1/2, 1/2}^{1/2, (FSI)}(\mathbf{p}_N, E) - \mathcal{O}_{-1/2, -1/2}^{1/2, (FSI)}(\mathbf{p}_N, E) + c.c. \right] \quad (6)$$

Indeed, the evaluation was carried out for the spectator SiDIS, where a deuteron is detected in the final state. Figure 1 illustrates the effects of FSI, evaluated within the GEA, by showing the absolute value of  $\mathcal{P}_{||}^{\frac{1}{2}}$ , for the above mentioned spectator SiDIS, as a function of the missing momentum,  $\mathbf{p}_{\text{mis}} \equiv \mathbf{P}_{\text{deut}}$ , in both the parallel ( $\theta_{\text{mis}} = 180^\circ$ ), and perpendicular ( $\theta_{\text{mis}} = 90^\circ$  and  $\phi_{\text{mis}} = 180^\circ$ ), kinematics. It is interesting to note that a wise choice of the kinematical variables can minimize the FSI effects, as already shown in the unpolarized case [7]. As shown in Fig. 1, FSI is negligible at low values of  $|\mathbf{p}_{\text{mis}}|$  (in this case one has a fast final debris, given  $\mathbf{p}_X \sim \mathbf{q}$ ), while it starts to be sizable for  $|\mathbf{p}_{\text{mis}}| \geq 1 \text{ fm}^{-1}$ , where the equal sign holds for the perpendicular kinematics (cf the right panel).

**Table 1**1) PWIA:  $p_n = 0.878$ ,  $p_p = -0.023$ ,  $\theta_e = 30^\circ$ ,  $\theta_\pi = 14^\circ$ 

$E_{beam},$ GeV	$x_{Bj}$	$\nu$ GeV	$p_\pi$ GeV/c	$f_n(x, z)$	$p_n f_n$	$f_p(x, z)$	$p_p f_p$
8.8	0.21	7.55	3.40	0.304	0.266	0.348	$-8.410^{-3}$
8.8	0.29	7.15	3.19	0.286	0.251	0.357	$-8.510^{-3}$
8.8	0.48	6.36	2.77	0.257	0.225	0.372	$-8.910^{-3}$
11	0.21	9.68	4.29	0.302	0.265	0.349	$-8.310^{-3}$
11	0.29	9.28	4.11	0.285	0.250	0.357	$-8.510^{-3}$

2) FSI:  $p_n = 0.756$ ,  $p_p = -0.027$ ,  $\langle \sigma_{eff} \rangle = 71 \text{ mb}$ 

$E_{beam},$ GeV	$x_{Bj}$	$\nu$ GeV	$p_\pi$ GeV/c	$f_n(x, z)$	$p_n f_n$	$f_p(x, z)$	$p_p f_p$
8.8	0.21	7.55	3.40	0.353	0.267	0.405	$-1.110^{-2}$
8.8	0.29	7.15	3.19	0.332	0.251	0.415	$-1.110^{-2}$
8.8	0.48	6.36	2.77	0.298	0.225	0.432	$-1.210^{-2}$
11	0.21	9.68	4.29	0.351	0.266	0.405	$-1.10^{-2}$
11	0.29	9.28	4.11	0.331	0.250	0.415	$-1.110^{-2}$

To illustrate the influence of the FSI effects on the extraction procedures of the neutron information, let us consider the preliminary comparison, for a JLAB12 kinematics [1], between the PWIA values of both dilution factors and nucleon polarizations that appear in Eq. (1) and the corresponding calculations with FSI effects taken into account. As shown in Table 1, FSI can sizably modify the overlaps given in Eq. (4), that are necessary for evaluating the quantities in Eq. (1). In particular, about a 15 – 20% depolarization effect of the nucleons in  $^3\text{He}$  is produced. It should be pointed that, even if both polarizations and dilution factors in Eq. (1) are affected by the presence of FSI, fortunately their product does not. Therefore the extraction procedure, can be safely applied. This seems a quite encouraging message, from the phenomenological point of view, and motivates the application of such an approach, combined with the the Poincaré covariant treatment of the nucleus tensor, to the extraction procedure of the neutron SSAs from the  $^3\text{He}$  SSAs, to be considered elsewhere [2].

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